METRIC PROPERTIES OF AN EXPANDED METAL PLANE STRUCTURE

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Key words. Expanded metal, frontier plane surface, frontier parabolical conical surface, hole

Abstract. The metric description of a structure consisting in an expanded metal plaque is given in the case of a standard expanding process by triangular knives.

1. Introduction and preliminary results

The domain of Expanded Metal is the most versatile one for more companies producing or using it. As consequence, many optimisation problems should be solved in connection with the manufacture and the use of this material. The metric properties of the structures made out of expanded metal are needed in this context. In this paper we deal with expanded metal plane standard structures, as in figure 1, studying them from metric point of view. They consist in a series of neighbouring cells, having their frontier made out of two types of surfaces: plane surfaces and conic surfaces of parabolic directory curve. We use the example illustrated in fgure 1, which is obtained by a standard expanding process using triangular knives.

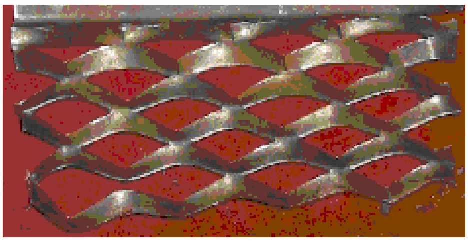

Fig. 1. Plane structure of expanded metal

The metric properties of this structure, which are studied, are: the area of the metal frontier of each cell, the area of the interior hole of each cell, the ratio between the initial surface of the material and the resulted expanded metal.

The other types of expanded metal are easy to study in an analogous manner.

In what follows, we use the following notations: h_x = the step on Ox axis, h_y = the step on Oy, h_z = the step on Oz and q = the semi-distance between the cells, along the directions parallel to the x - axis. The elements h_x, h_y and q are positive real numbers, while h_z is a negative real number, with respect to the three dimensional coordinates system, as in figure 2. The physical significance of these numbers is:

h_x = the length of a cut plus the length of a bridge between two neighbouring cuts;
h_y = the distance between two rows of successive cuts;

h_z = the vertical distance of penetration of the knife during two successive cuts.
q = half of the distance between two successive cuts, located on the same cuts row.
The graphical representation of the plane plaque submitted to expanding is in figure 2.

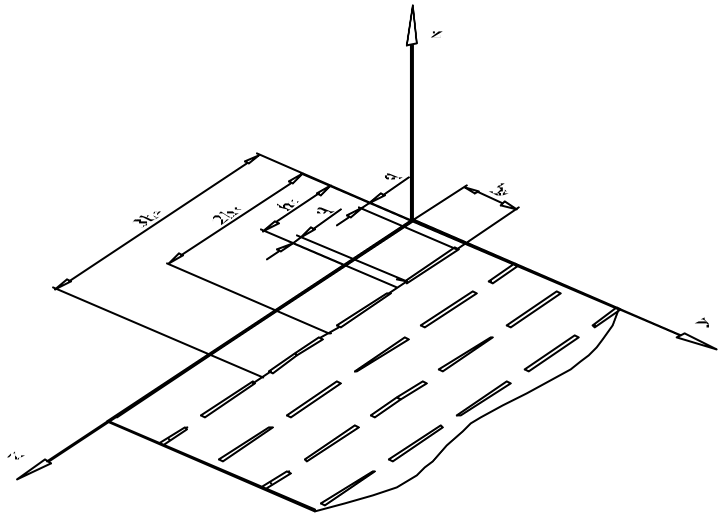

Fig. 2 The plane plaque submitted to the expanding process

In the following, we take into account only one cell resulted after expanding the plaque in figure 2, which results after cutting the straight line segment having the coordinates $((j\text{-}1)h_x, (k+1)h_y)$ and $(jh_x, (k+1)h_y)$ in the plane $z = 0$. Figure 3 depicts this cell.

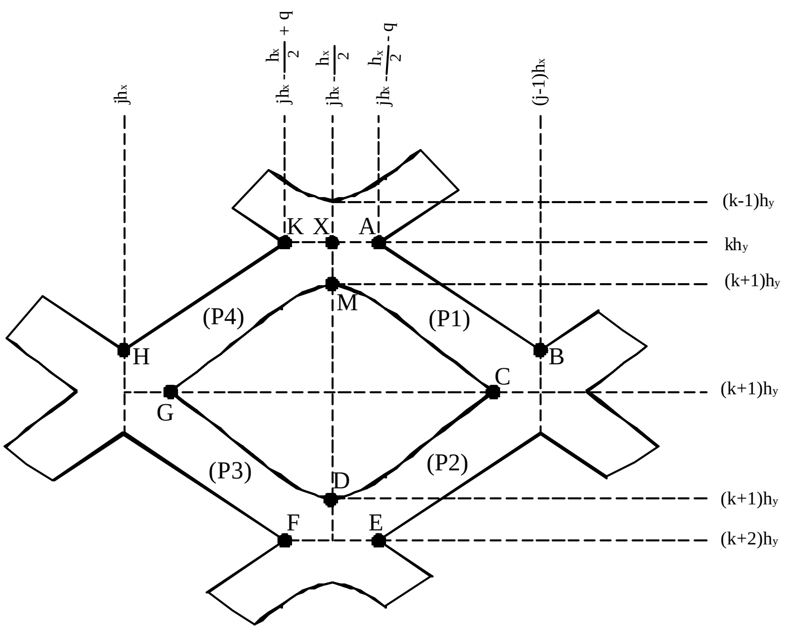

Fig. 3. The expanded metal cell

Consider that, after cutting, the point of x - coordinate $x = jh_x - \frac{h_x}{2}$ from the row having the ordinate $y = kh_y$, has vertically moved with $u = \frac{h_z}{2}$ units, such as the cut on the line $y = (k+1)h_y$ will result in the descent of the point $x = jh_x - \frac{h_x}{2}$ with $h_z - u = \frac{h_z}{2}$ more units.

The cell, which results after expanding, depicted in figure 3, is limited by four planes, denoted by P_1 = (ABC), P_2 = (CDE), P_3 = (DFG) and P_4 = (GHK). Their equations and their directions are derived in the following property from [2].

Property 1.1. *The equations of the plane surfaces containing the frontier of the expanded metal cell in figure 3 are:*

$$(P_1):\ -\frac{h_z h_y}{2}\cdot x + \frac{h_z q}{2}\cdot y + h_y\left(q - \frac{h_x}{2}\right)\cdot z + \frac{-1-2k}{2}h_y h_z q + \frac{2j+k-1}{4}h_x h_y h_z = 0,$$

$$(P_2):\ -\frac{h_y h_z}{2}\cdot x - \frac{h_z q}{2}\cdot y + h_y\left(\frac{h_x}{2} - q\right)\cdot z + \frac{2j-k-2}{4}h_x h_y h_z + \frac{2k+3}{2}q h_y h_z = 0,$$

$$(P_3):\ -\frac{h_z h_y}{2}\cdot x + \frac{h_z q}{2}\cdot y - h_y\left(\frac{h_x}{2} - q\right)\cdot z + \frac{2j+k+1}{4}h_x h_y h_z - \frac{2k+3}{2}q h_y h_z = 0,$$

$$(P_4):\ -\frac{h_y h_z}{2}\cdot x - \frac{q h_z}{2}\cdot y + h_y\left(\frac{h_x}{2} - q\right)\cdot z + \frac{2k+1}{2}q h_y h_z + \frac{2j-k-1}{4}h_x h_y h_z = 0,$$

$$(P_5):\ z = \frac{k}{2}h_z.$$

The cell is also limited by a parabolic conic surface, having the vertex in A and the directory curve the parabola ? , which have been analytically described in [2], as:

Property 1.2. *The equation of the parabola having the vertex in* M *and containing the points* C *and* G *is:*

$$?:\ \begin{cases} z = \dfrac{2(k+1)h_z}{(2q-h_x)^2 + 2kh_z}\left[\left(x - jh_x + \dfrac{h_x}{2}\right)^2 + \dfrac{k}{2}h_z\right]. \\ y = (k+1)h_y \end{cases}$$

Property 1.3. *The conic surface having the vertex A and parabola ? as directory curve is represented by the equation:*

$$\frac{h_y\left(z - \frac{k}{2}h_z\right)}{y - kh_y} - \frac{\frac{k+1}{2}h_z}{\left(q - \frac{h_x}{2}\right)^2 + \frac{k}{2}h_z}\left(\frac{x - jh_x + \frac{h_x}{2} + q}{y - kh_y}h_y - q\right)^2 = \frac{\frac{k}{2}h_z^2 - \frac{k}{2}\left(q - \frac{h_x}{2}\right)^2 h_z}{\left(q - \frac{h_x}{2}\right)^2 + \frac{k}{2}h_z}.$$

The previous properties are the basic ones for the metric approach of the structure.

2. Metric properties of the standard expanded metal structure

The following basic formulas (see [1], [4]) will be used in order to compute the metric elements of a cell as in figure 3.

The area of a triangle denoted by ABC is:

$$S^2(ABC)=\frac{1}{4}\left(\begin{vmatrix} y_A & z_A & 1 \\ y_B & z_B & 1 \\ y_C & z_C & 1 \end{vmatrix}^2+\begin{vmatrix} x_A & z_A & 1 \\ x_B & z_B & 1 \\ x_C & z_C & 1 \end{vmatrix}^2+\begin{vmatrix} x_A & y_A & 1 \\ x_B & y_B & 1 \\ x_C & y_C & 1 \end{vmatrix}^2\right) \quad (2.1)$$

The area of the parabolic cone having the explicit equation $z = f(x,y)$ is given by:

$$S(AMC)=\iint\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dxdy \quad (2.2)$$

The frontier of a cell is the union of four congruent bars. The area of the frontier of a cell is:

$$A(cell)=4\left[S(ABC)+2S(AXM)+S(AMC)\right]. \quad (2.3)$$

Here, the expression in brackets represents the area of a bar, which is one of the sides of the cell from figure 3. This is obtained by adding together the areas of the plane triangles ABC, AMX and the three dimensional triangle AMC with a parabolic side, represented in figure 4.

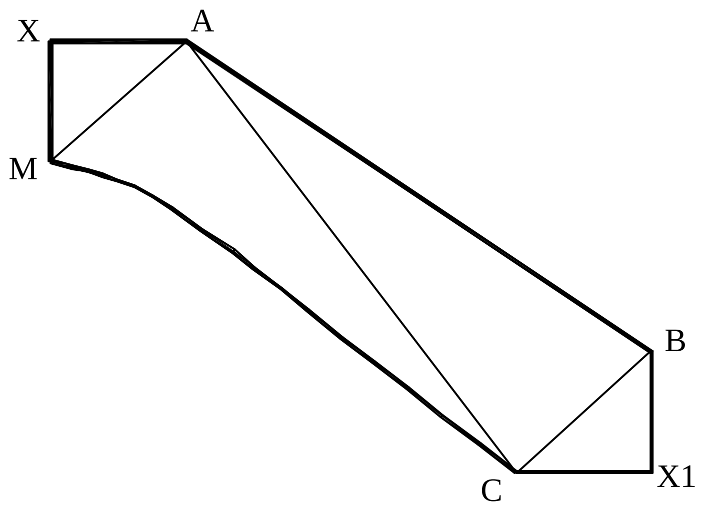

Fig. 4. Side of a cell

Consider k = 1 and j = 1, since all the cells are metrically identical, resulting:

$$A\left(\frac{h_x}{2}-q,\quad h_y,\quad \frac{h_z}{2}\right),$$
$$B\left(0,\quad h_y,\quad h_z\right),$$
$$C\left(q,\quad 2h_y,\quad h_z\right),$$
$$M\left(\frac{h_x}{2},\quad 2h_y,\quad \frac{h_z}{2}\right),$$
$$X\left(\frac{h_x}{2},\quad h_y,\quad \frac{h_z}{2}\right).$$

In this case, using (2.1), one has:

$$S^2(ABC)=\frac{1}{4}\left(\begin{vmatrix}\frac{h_x}{2}-q & \frac{h_z}{2} & 1\\ 0 & h_z & 1\\ q & h_z & 1\end{vmatrix}^2+\begin{vmatrix}h_y & \frac{h_z}{2} & 1\\ h_y & h_z & 1\\ 2h_y & h_z & 1\end{vmatrix}^2+\begin{vmatrix}\frac{h_x}{2}-q & h_y & 1\\ 0 & h_y & 1\\ q & 2h_y & 1\end{vmatrix}^2\right).$$

Therefore, the area of triangle ABC is:

$$S(ABC)=\frac{1}{4}\sqrt{h_z^2q^2+h_y^2h_z^2+h_x^2h_y^2-8h_xh_y^2q+4h_y^2q^2} \qquad (2.4)$$

Similarly, the area of triangle AXM is:

$$S^2(AXM)=\frac{1}{4}\left(\begin{vmatrix}h_y & \frac{h_z}{2} & 1\\ 2h_y & \frac{h_z}{2} & 1\\ h_y & \frac{h_z}{2} & 1\end{vmatrix}^2+\begin{vmatrix}\frac{h_x}{2}-q & \frac{h_z}{2} & 1\\ \frac{h_x}{2} & \frac{h_z}{2} & 1\\ \frac{h_x}{2} & \frac{h_z}{2} & 1\end{vmatrix}^2+\begin{vmatrix}\frac{h_x}{2}-q & h_y & 1\\ \frac{h_x}{2} & 2h_y & 1\\ \frac{h_x}{2} & h_y & 1\end{vmatrix}^2\right).$$

Therefore, the area of AXM is:

$$S(AXM)=\frac{1}{2}h_yq \qquad (2.5)$$

The area of the 3D triangle having a parabolic side generated by the parabolic cone from property 1.3 needs the explicit equation of this surface. For k =1 and j= 1, property 1.3 becomes:

$$z(x,\quad y)=\frac{y-h_y}{h_y}\left[\frac{\frac{1}{2}h_z^2-\frac{1}{2}\left(q-\frac{1}{2}h_x\right)h_z}{\left(q-\frac{1}{2}h_x\right)^2+\frac{1}{2}h_z}+\frac{h_z\left(\frac{x-\frac{1}{2}h_x+q}{y-h_y}h_{y-q}\right)^2}{\left(q-\frac{1}{2}h_x\right)^2+\frac{1}{2}h_z}+\frac{\frac{1}{2}h_y h_z}{y-h_y}\right].$$

The partial derivatives of this function are:

$$\frac{\partial z}{\partial x}=\frac{2h_z\left(\frac{x-\frac{1}{2}h_x+q}{y-h_y}-q\right)}{h_y\left[\left(q-\frac{h_x}{2}\right)^2+\frac{1}{2}h_z\right]}, \tag{2.6}$$

$$\frac{\partial z}{\partial y}=\frac{\frac{1}{2}h_z^2-\frac{1}{2}\cdot\left(q-\frac{h_x}{2}\right)h_z+q^2h_z-\frac{h_y h_z\cdot\left(x-\frac{1}{2}h_x+q\right)^2}{\left(y-h_y\right)^2}}{h_y\left[\left(q-\frac{h_x}{2}\right)^2+\frac{1}{2}h_z\right]}. \tag{2.7}$$

Replacing (2.6) and (2.7) in (2.2) and computing the integral, one gets the area of the conic surface AMC:

$$S(AMC)=\frac{1}{4}\sqrt{h_z^2q^2+h_y^2h_z^2+h_x^2h_y^2-8h_xh_y^2q+4h_y^2q^2}+\frac{1}{2}h_yq \tag{2.8}$$

Thus, replacing (2.4), (2.5) and (2.8) in (2.3), one obtains the following:

Property 2.1. *The area of the frontier of a cell of an expanded metal structure is:*

$$A(cell)=2\sqrt{h_z^2q^2+h_y^2h_z^2+h_x^2h_y^2-8h_xh_y^2q+4h_y^2q^2}+6h_yq. \tag{2.9}$$

The area of the hole of a cell equals to the double of the area of the curved triangle MCD from figure 4. Also, it is approximated, with a negligible error, by the double of the area of triangle CDG. For k = 1 and j = 1, one has

$$C(q,\quad 2h_y,\quad h_z),$$

$$D\left(\frac{h_x}{2},\quad 2h_y,\quad \frac{3h_z}{2}\right),$$

$$G(h_x-q,\quad 2h_y,\quad h_z).$$

The area of CDG is given by:

$$S^2(CDG)=\frac{1}{4}\left(\begin{vmatrix}2h_y & h_z & 1\\ 2h_y & \frac{3}{2}h_z & 1\\ 2h_y & h_z & 1\end{vmatrix}^2+\begin{vmatrix}q & h_z & 1\\ \frac{h_x}{2} & \frac{3}{2}h_z & 1\\ h_x-q & h_z & 1\end{vmatrix}^2+\begin{vmatrix}q & 2h_z & 1\\ \frac{h_x}{2} & 2h_z & 1\\ h_x-q & 2h_z & 1\end{vmatrix}^2\right).$$

The computation gives now:

$$S(CDG)=\frac{1}{2}\left|h_z q-\frac{1}{2}h_x h_z\right| . \qquad (2.10)$$

Property 2.2. *The area of the hole of one cell is*:

$$A_g=h_z\left|q-\frac{1}{2}h_x\right|. \qquad (2.11)$$

Now, it is possible to compute the total area of a structure containing *n* cells and *p* fragments of cells in the neighbourhood of its frontier. Also, it is easy to express the relationship between the original area of the material and the resulted area after the expanding process.

The metric formulas derived in this paper are useful to approach various problems of shape optimisation. Also, this description of the grid allows us to perform detailed studies of its physical properties.

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